Chapter 2 Feynman Path Integral Formulation Springer

- Wherever possible simple examples, which illustrate the main ideas, are provided before embarking on the actual discussion of the problem of interest - The book introduces the readers to problems of great current interest, like instantons, calorons, vortices, magnetic monopoles - QCD at finite temperature is discussed at great length, both in perturbation theory and in Monte Carlo simulations - The book contains many figures showing numerical results of pioneering work
Providing a self-contained step-by-step explanation, this book provides a guide to path integral methods for readers with a basic knowledge of quantum mechanics
Advances in Quantum Chemistry
The path integral approach has proved extremely useful for the understanding of the most complex problems in quantum field theory, cosmology, and condensed matter physics. Path Integrals in Physics: Volume II, Quantum Field Theory, Statistical Physics and other Modern Applications covers the fundamentals of path integrals, both the Wiener and Feynman types, and their many applications in physics. The book deals with systems that have an infinite number of degrees of freedom. It discusses the general physical background and concepts of the path integral approach used, followed by a detailed presentation of the most typical and important applications as well as problems with either their solutions or hints how to solve them. Each chapter is self-contained and can be considered as an independent textbook. It provides a comprehensive, detailed, and systematic account of the subject suitable for both students and experienced researchers.
Action and Symmetries

Path Integrals in Physics
The Feynman Path Integral
The Representation-Independent Propagators for General Lie Groups
Advances in Quantum Chemistry

This book provides an ideal introduction to the use of Feynman path integrals in the fields of quantum mechanics and statistical physics. It is written for graduate students and researchers in physics, mathematical physics, applied mathematics as well as chemistry. The material is presented in an accessible manner for readers with little knowledge of quantum mechanics and no prior exposure to path integrals. It begins with elementary concepts and a review of quantum mechanics that gradually builds the framework for the Feynman path integrals and how they are applied to problems in quantum mechanics and statistical physics. Problem sets throughout the book allow readers to test their understanding and reinforce the explanations of the theory in real situations. Features: Comprehensive and rigorous yet, presents an easy-to-understand approach. Applicable to a wide range of disciplines. Accessible to those with little, or basic, mathematical understanding.
The Feynman Path IntegralExplained and Derived for Quantum Electrodynamics and Quantum Field Theory
Graduate-level, systematic presentation of path integral approach to calculating transition elements, partition functions, and source functionals. Covers Grassmann variables, field and gauge field theory, perturbation theory, and nonperturbative results. 1992 edition.
First Published in 2018. Routledge is an imprint of Taylor & Francis, an Informa company.
Lattice Gauge Theories
Path Integral Methods
Path Integrals and Hamiltonians
An Introduction to Gauge Theories
The Theory of Quark and Gluon Interactions
Mathematical Feynman Path Integrals And Their Applications (Second Edition)

Suitable for advanced undergraduates and graduate students, this text develops the techniques of path integration and deals with applications, covering a host of illustrative examples. 26 figures. 1981 edition.
This book is the first to cover marketing management issues in geographically remote industrial clusters (GRICs). The phenomena of GRICs have increased in importance, especially in the Nordic countries, due to changes in industry structures as well as political ambitions. The practice of marketing and marketing management is not singular to industry clusters in Nordic countries. Remote areas in parts of the United States, South and Central America, and South East Asia exhibit similar tendencies.The problems faced by many entrepreneurial managers managing start-up or even existing enterprises are complex and require an in-depth understanding not only of the problems themselves, but also of the contextual framework in which these problems need to be solved. This book contains original cases that cover issues like cluster formation, information gathering, marketing strategies and operations, and information-technology. Examples come from industries like textile & furniture, automobile, agro-machinery, food, wine, software, and management consulting.
This text on quantum mechanics begins by covering all the main topics of an introduction to the subject. It then concentrates on newer developments. In particular it continues with the perturbative solution of the Schrödinger equation for various potentials and thereafter with the introduction and evaluation of their path integral counterparts. Considerations of the large order behavior of the perturbation expansions show that in most applications these are asymptotic expansions. The parallel consideration of path integrals requires the evaluation of these around periodic classical configurations, the fluctuation equations about which lead back to specific wave equations. The period of the classical configurations is related to temperature, and permits transitions to the thermal domain to be classified as phase transitions. In this second edition of the text important applications and numerous examples have been added. In particular, the chapter on the Coulomb potential has been extended to include an introduction to chemical bonds, the chapter on periodic potentials has been supplemented by a section on the band theory of metals and semiconductors, and in the chapter on large order behavior a section has been added illustrating the success of converging factors in the evaluation of asymptotic expansions. Detailed calculations permit the reader to follow every step.
The special Internet categories are: Physics; Engineering Quantum Physics; and Applied Mathematics. The emphasis in this monograph is on non-trivial path integral variable change on previously obtained path integral solutions for difficult stochastic and functional equations by keeping the main objective to arrive at...another path integral which the author expects to be in a 'final' suitable form of become predictive. Note that path-integrals are mathematical objects specially tailored to the work of our modern 'slavers': computer machines.
Quantum Field Theory in Condensed Matter Physics
Techniques and Applications of Path Integration
Path Integration: Trieste 1991, Lectures On - Proceedings Of The Adriatico Research Conference
Path Integrals, Hyperbolic Spaces and Selberg Trace Formulae
Rigorous Time Slicing Approach to Feynman Path Integrals
Principles, Methods, and Case Studies

This is an approachable introduction to the important topics and recent developments in the field of condensed matter physics. First, the general language of quantum field theory is developed in a way appropriate for dealing with systems having a large number of degrees of freedom. This paves the way for a description of the basic processes in such systems. Applications include various aspects of superfluidity and superconductivity, as well as a detailed description of the fractional quantum Hall liquid.
First published in 1983, this book has become a classic among advanced textbooks. The new fourth edition maintains the high standard of its predecessors. The book offers basic knowledge of field theory and particle phenomenology. The author presents the basic facts of quark and gluon physics in pedagogical form. Explanations of theory are supported throughout with experimental findings. The text provides readers with sufficient understanding to follow modern research articles. This fourth edition presents a new section on heavy quark effective theories, more material on lattice QCD and on chiral perturbation theory.
In this second edition, a comprehensive review is given for path integration in two- and three-dimensional (homogeneous) spaces of constant and non-constant curvature, including an enumeration of all the corresponding coordinate systems which allow separation of variables in the Hamiltonian and in the path integral. The corresponding path integral solutions are presented as a tabulation. Proposals concerning interbasis expansions for spheroidal coordinate systems are also given. In particular, the cases of non-constant curvature Darboux spaces are new in this edition. The volume also contains results on the numerical study of the properties of several integrable billiard systems in compact domains (i.e. rectangles, parallelepipeds, circles and spheres) in two- and three-dimensional flat and hyperbolic spaces. In particular, the discussions of integrable billiards in circles and spheres (flat and hyperbolic spaces) and in three dimensions are new in comparison to the first edition. In addition, an overview is presented on some recent achievements in the theory of the Selberg trace formula on Riemann surfaces, its super generalization, their use in mathematical physics and string theory, and some further results derived from the Selberg (super-) trace formula. Contents:IntroductionPath Integrals in Quantum MechanicsSeparable Coordinate Systems on Spaces of Constant CurvaturePath Integrals in Pseudo-Euclidean GeometryPath Integrals in Euclidean SpacesPath Integrals on SpheresPath Integrals on HyperboloidsPath Integral on the Complex SpherePath Integrals on Hermitian Hyperbolic SpacePath Integrals on Darboux SpacesPath Integrals on Single-Sheeted HyperboloidsMiscellaneous Results on Path IntegrationBilliard Systems and Periodic Orbit TheoryThe Selberg Trace FormulaThe Selberg Super-Trace FormulaSummary and Discussion Readership: Graduate and researchers in mathematical physics. Keywords:Path Integrals;Selberg Trace Formula;Quantum Chaos;Coordinate Systems;Homogeneous Spaces;Spaces of Non-Constant Curvature;Separation of VariablesKey Features:The 2nd edition brings the text up to date with new developments and results in the fieldContains enumeration of many explicit path integrals solutionsReviews: "This book is a good survey of results in a fascinating, highly geometrical, field in which much remains to be done." Zentralblatt MATH
Every part of physics offers examples of non-stability phenomena, but probably nowhere are they so plentiful and worthy of study as in the realm of quantum theory. The present volume is devoted to this problem: we shall be concerned with open quantum systems, i.e. those that cannot be regarded as isolated from the rest of the physical universe. It is a natural framework in which non-stationary processes can be investigated. There are two main approaches to the treatment of open systems in quantum theory. In both the system under consideration is viewed as part of a larger system, assumed to be isolated in a reasonable approximation. They are differentiated mainly by the way in which the state Hilbert space of the open system is related to that of the isolated system - either by orthogonal sum or by tensor product. Though often applicable simultaneously to the same physical situation, these approaches are complementary in a sense and are adapted to different purposes. Here we shall be concerned with the first approach, which is suitable primarily for a description of decay processes, absorption, etc. The second approach is used mostly for the treatment of various relaxation phenomena. It is comparably better examined at present; in particular, the reader may consult a monograph by E. B. Davies.
An Introduction
Open Quantum Systems and Feynman Integrals
Path-integral methods and their applications
The Feynman Integral and Feynman's Operational Calculus
SchrÃ¶dinger Equation and Path Integral Second Edition
The New Millennium Edition: Mainly Electromagnetism and Matter

Looks at quantum mechanics, covering such topics as perturbation method, statistical mechanics, path integrals, and quantum electrodynamics.
The aim of this book is to derive the Feynman Path Integral from first principles and apply it to a simple system, before demonstrating its equivalence to the Schrodinger formulation of quantum mechanics. The necessary prerequisite knowledge makes it suitable for undergraduate or graduate level physicists.Each step is detailed and every calculation is performed explicitly, so they may be followed with ease. Many of the detailed calculations are also hand-written to avoid any ambiguity associated with technical formatting.Chapter 1 gives an introduction to the Feynman Path Integral and the reason for its use. Chapter 2 then summarises the relevant quantum mechanics, including using the operator method to calculate the energy states for the simple harmonic oscillator. Chapter 3 introduces the action functional for an elementary system before Lagrangian and Hamiltonian mechanics are explained in Chapter 4, including canonical transformations and generating functions. The Feynman Path Integral is then derived in Chapter 5, with calculation of the transition amplitude for a particle to move between two fixed points in a given time. Chapter 6 then applies the Feynman Path Integral to the forced harmonic oscillator. In Chapter 7, we apply the path integral to a discrete, Euclidean time action and then in Chapter 8, use the path integral to calculate the ground and successive state energies for the simple harmonic oscillator. In Chapter 9, the general equivalence of the Path Integral and Schrodinger formulations of quantum mechanics are demonstrated. Finally, in Chapter 10, there is a short narrative on the application to Quantum Electrodynamics and Quantum Field Theory in general.
The Feynman path integrals are becoming increasingly important in the applications of quantum mechanics and field theory. The path integral formulation of quantum anomalies, i.e. the quantum breaking of certain symmetries, can now cover all the known quantum anomalies in a coherent manner. In this book the authors provide an introduction to the path integral method in quantum field theory and its applications to the analyses of quantum anomalies. No previous knowledge of field theory beyond advanced undergraduate quantum mechanics is assumed. The book provides the first coherent introductory treatment of the path integral formulation of chiral and Weyl anomalies, with applications to gauge theory in two and four dimensions, conformal field theory and string theory. Explicit and elementary path integral calculations of most of the quantum anomalies covered are given. The conceptual basis of the path integral bosonization in two-dimensional theory, which may have applications to condensed matter theory, for example, is clarified. The book also covers the recent interesting developments in the treatment of fermions and chiral anomalies in lattice gauge theory.
The quantization of physical systems moving on group and symmetric spaces has been an area of active research over the past three decades. This book shows that it is possible to introduce a representation-independent propagator for a real, separable, connected and simply connected Lie group with irreducible, square-integrable representations. For a given set of kinematical variables this propagator is a single generalized function independent of any particular choice of fiducial vector and the irreducible representations of the Lie group generated by these kinematical variables, which nonetheless correctly propagates each element of a continuous representation based on the coherent states associated with these kinematical variables. Furthermore, the book shows that it is possible to construct regularized lattice phase space path integrals for a real, separable, connected and simply connected Lie group with irreducible, square-integrable representations, and although the configuration space is in general a multidimensional curved manifold, it is shown that the resulting lattice phase space path integral has the form of a lattice phase space path integral on a multidimensional flat manifold. Hence, a novel and extremely natural phase space path integral quantization is obtained for general physical systems whose kinematical variables are the generators of a connected and simply connected Lie group. This novel phase space path integral quantization is (a) exact, (b) more general than, and (c) free from the limitations of the previously considered path integral quantizations of free physical systems moving on group manifolds. To illustrate the general theory, a representation-independent propagator is explicitly constructed for SU(2) and the affine group. Contents:Mathematical PreludePhysical PreludeA Review of Some Means to Define Path Integrals on Group and Symmetric SpacesNotations and PreliminariesThe Representation Independent Propagator for a General Lie GroupClassical Limit of the Representation Independent PropagatorConclusion and OutlookContinuous Representation TheoryExact Lattice Calculations Readership: Physicists. Keywords:Global Analysis;Analysis on Manifolds [For Geometric Integration Theory];Spaces and Manifolds of Mappings;Quantum Mechanics (Feynman Path Integrals), Relativity, Fluid Dynamics;Quantum Theory;General Quantum Mechanics and Problems of Quantization;Path IntegralsReviews: "The author explains the theory clearly and the book is almost self-contained ..." Contemporary Physics
Quantum Mechanics and Path Integrals [by] R. P. Feynman [and] A. R. Hibbs
Methods of Bosonic Path Integrals Representations
Continuous Quantum Measurements and Path Integrals
An Introduction to Quantum Theory
Functional Integration
An Introduction To Quantum Theory, Second Edition

This book explains key concepts in theoretical chemistry and explores practical applications in structural chemistry. For experimentalists, it highlights concepts that explain the underlying mechanisms of observed phenomena, and at the same time provides theoreticians with explanations of the principles and techniques that are important in property design. Themes covered include conceptual and applied wave functions and density functional theory (DFT) methods, electronegativity and hard and soft (Lewis) acid and base (HSAB) concepts, hybridization and aromaticity, molecular magnetism, spin transition and thermochromism. Offering insights into designing new properties in advanced functional materials, it is a valuable resource for undergraduates of physical chemistry, cluster chemistry and structure/reactivity courses as well as graduates and researchers in the fields of physical chemistry, chemical modeling and functional materials.

This book provides the most comprehensive mathematical treatment to date of the Feynman path integral and Feynman's operational calculus. It is accessible to mathematicians, mathematical physicists and theoretical physicists. Including new results and much material previously only available in the research literature, this book discusses both the mathematics and physics background that motivate the study of the Feynman path integral and Feynman's operational calculus, and also provides more detailed proofs of the central results.

Covering the theory of computation, information and communications, the physical aspects of computation, and the physical limits of computers, this text is based on the notes taken by one of its editors, Tony Hey, on a lecture course on computation given b

New edition features improved typography, figures and tables, expanded indexes, and 885 new corrections.

Explained and Derived for Quantum Electrodynamics and Quantum Field Theory

Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets

Path Integrals From Pev To Tev: 50 Years After Feynman's Paper - Proceedings Of The Sixth International Conference

Field Theory : A Path Integral Approach

Lectures On Computation

Quantum Mechanics and Path Integrals

Advances in technology are taking the accuracy of macroscopic as well as microscopic measurements close to the quantum limit, for example, in the attempts to detect gravitational waves. Interest in continuous quantum measurements has therefore grown considerably in recent years. Continuous Quantum Measurements and Path Integrals examines these measurements using Feynman path integrals. The path integral theory is developed to provide formulae for concrete physical effects. The main conclusion drawn from the theory is that an uncertainty principle exists for processes, in addition to the familiar one for states. This implies that a continuous measurement has an optimal accuracy-a balance between inefficient error and large quantum fluctuations (quantum noise). A well-known expert in the field, the author concentrates on the physical and conceptual side of the subject rather than the mathematical.

Feynman path integrals are ubiquitous in quantum physics, even if a large part of the scientific community still considers them as a heuristic tool that lacks a sound mathematical definition. Our book aims to refute this prejudice, providing an extensive and self-contained description of the mathematical theory of Feynman path integration, from the earlier attempts to the latest developments, as well as its applications to quantum mechanics.This second edition presents a detailed discussion of the general theory of complex integration on infinite dimensional spaces, providing on one hand a unified view of the various existing approaches to the mathematical construction of Feynman path integrals and on the other hand a connection with the classical theory of stochastic processes. Moreover, new chapters containing recent applications to several dynamical systems have been added.This book bridges between the realms of stochastic analysis and the theory of Feynman path integration. It is accessible to both mathematicians and physicists.

Introduces the powerful and flexible combination of Hamiltonian operators and path integrals in quantum mathematics.

This book proves that Feynman's original definition of the path integral actually converges to the fundamental solution of the Schrödinger equation at least in the short term if the potential is differentiable sufficiently many times and its derivatives of order equal to or higher than two are bounded. The semi-classical asymptotic formula up to the second term of the fundamental solution is also proved by a method different from that of Birkhoff. A bound of the remainder term is also proved.The Feynman path integral is a method of quantization using the Lagrangian function, whereas Schrödinger's quantization uses the Hamiltonian function. These two methods are believed to be equivalent. But equivalence is not fully proved mathematically, because, compared with Schrödinger's method, there is still much to be done concerning rigorous mathematical treatment of Feynman's method. Feynman himself defined a path integral as the limit of a sequence of integrals over finite-dimensional spaces which is obtained by dividing the time interval into small pieces. This method is called the time slicing approximation method or the time slicing method.This book consists of two parts. Part I is the main part. The time slicing method is performed step by step in detail in Part I. The time interval is divided into small pieces. Corresponding to each division a finite-dimensional integral is constructed following Feynman's famous paper. This finite-dimensional integral is not absolutely convergent. Owing to the assumption of the potential, it is an oscillatory integral. The oscillatory integral techniques developed in the theory of partial differential equations are applied to it. It turns out that the finite-dimensional integral gives a finite definite value. The stationary phase method is applied to it. Basic properties of oscillatory integrals and the stationary phase method are explained in the book in detail.Those finite-dimensional integrals form a sequence of approximation of the Feynman path integral when the division goes finer and finer. A careful discussion is required to prove the convergence of the approximate sequence as the length of each of the small subintervals tends to 0. For that purpose the book uses the stationary phase method of oscillatory integrals over a space of large dimension, of which the detailed proof is given in Part II of the book. By virtue of this method, the approximate sequence converges to the limit. This proves that the Feynman path integral converges. It turns out that the convergence occurs in a very strong topology. The fact that the limit is the fundamental solution of the Schrödinger equation is proved also by the stationary phase method. The semi-classical asymptotic formula naturally follows from the above discussion.A prerequisite for readers of this book is standard knowledge of functional analysis. Mathematical techniques required here are explained and proved from scratch in Part II, which occupies a large part of the book, because they are considerably different from techniques usually used in treating the Schrödinger equation.

Path Integrals and Quantum Anomalies

Monte Carlo evaluation of Feynman path integrals in imaginary time

The Feynman Lectures on Physics, Vol. II

Random Systems in Classical Physics

Volume II Quantum Field Theory, Statistical Physics and other Modern Applications

Quantum Many-particle Systems

Functional integration successfully entered physics as path integrals in the 1942 PhD dissertation of Richard P. Feynman, but it made no sense at all as a mathematical definition. Cartier and DeWitt-Morette have created, in this book, a fresh approach to functional integration. The book is self-contained: mathematical ideas are introduced, developed, generalised and applied. In the authors' hands, functional integration is shown to be a robust, user-friendly and multi-purpose tool that can be applied to a great variety of situations, for example: systems of indistinguishable particles; Aharonov–Bohm systems; supersymmetry; non-gaussian integrals. Problems in quantum field theory are also considered. In the final part the authors outline topics that can be profitably pursued using material already presented.

This book explains the fundamental concepts and theoretical techniques used to understand the properties of quantum systems having large numbers of degrees of freedom. A number of complimentary approaches are developed, including perturbation theory; nonperturbative approximations based on functional integrals; general arguments based on order parameters, symmetry, and Fermi liquid theory; and stochastic methods.

This volume contains revised and extended research articles written by prominent researchers. Topics covered include electrical engineering, circuits, artificial intelligence, data mining, imaging engineering, bioinformatics, internet computing, software engineering, and industrial applications. The book offers tremendous state-of-the-art advances in electrical engineering and also serves as an excellent reference work for researchers and graduate students working with/on electrical engineering.

This is the fourth, expanded edition of the comprehensive textbook published in 1990 on the theory and applications of path integrals. It is the first book to explicitly solve path integrals of a wide variety of nontrivial quantum-mechanical systems, in particular the hydrogen atom. The solutions have become possible by two major advances. The first is a new euclidean path integral formula which increases the restricted range of applicability of Feynman's famous formula to include singular attractive 1/r and 1/r2 potentials. The second is a simple quantum equivalence principle governing the transformation of euclidean path integrals to spaces with curvature and torsion, which leads to time-sliced path integrals that are manifestly invariant under coordinate transformations. In addition to the time-sliced definition, the author gives a perturbative definition of path integrals which makes them invariant under coordinate transformations. A consistent implementation of this property leads to an extension of the theory of generalized functions by defining uniquely integrals over products of distributions. The powerful Feynman–Kleinert variational approach is explained and developed systematically into a variational perturbation theory which, in contrast to ordinary perturbation theory, produces convergent expansions. The convergence is uniform from weak to strong couplings, opening a way to precise approximate evaluations of analytically unsolvable path integrals. Tunneling processes are treated in detail. The results are used to determine the lifetime of supercurrents, the stability of metastable thermodynamic phases, and the large-order behavior of perturbation expansions. A new variational treatment extends the range of validity of previous tunneling theories from large to small barriers. A corresponding extension of large-order perturbation theory also applies now to small orders. Special attention is devoted to path integrals with topological restrictions. These are relevant to the understanding of the statistical properties of elementary particles and the entanglement phenomena in polymer physics and biophysics. The Chern–Simons theory of particles with fractional statistics (anyons) is introduced and applied to explain the fractional quantum Hall effect. The relevance of path integrals to financial markets is discussed, and improvements of the famous Black–Scholes formula for option prices are given which account for the fact that large market fluctuations occur much more frequently than in the commonly used Gaussian distributions. The author's other book on ‘Critical Properties of ?4 Theories’ gives a thorough introduction to the field of critical phenomena and develops new powerful resummation techniques for the extraction of physical results from the divergent perturbation expansions.

Principles and Methods

Structural Chemistry

Fourth Edition

Feynman Path Integrals in Quantum Mechanics and Statistical Physics

Introduction to Quantum Mechanics

Path Integrals and Quantum Processes

Written by world-leading experts in particle physics, this new book from Luciano Maiani and Omar Benhar, with contributions from the late Nicola Cabibbo, is based on Feynman's path integrals. Key elements of gauge theories are described—Feynman diagrams, gauge-fixing, Faddeev-Popov ghosts—as well as renormalization in Quantum Electrodynamics. Quarks and QCD interactions are introduced. Renormalization group and high momentum behaviour of the coupling constants is discussed in QED and QCD, with asymptotic freedom derived at one-loop. These concepts are related to the Higgs boson and models of grand unification. "... an excellent introduction to the quantum theory of gauge fields and their applications to particle physics. ... It will be an excellent book for the serious student and a good reference for the professional practitioner. Let me add that, scattered through the pages, we can find occasional traces of Nicola Cabibbo's style." —John Iliopoulos, CNRS-Ecole Normale Supérieure " ... The volume ends with an illuminating description of the expectation generated by the recent discovery of the Higgs boson, combined with the lack of evidence for super-symmetric particles in the mass range 0.6-1 TeV." —Arturo Menchaca-Rocha, FinstP, Professor of Physics, Mexico's National Autonomous University, Former President of the Mexican Academy of Sciences, Presidential Advisor "...The reader is masterfully guided through the subtleties of the quantum field theory and elementary particle physics from simple examples in Quantum Mechanics to salient details of modern theory." —Mikhail Voloshin, Professor of Physics, University of Minnesota

The 2nd edition of LNM 523 is based on the two first authors' mathematical approach of this theory presented in its 1st edition in 1976. An entire new chapter on the current forefront of research has been added. Except for this new chapter and the correction of a few misprints, the basic material and presentation of the first edition has been maintained. At the end of each chapter the reader will also find notes with further bibliographical information.

This is the third, significantly expanded edition of the comprehensive textbook published in 1990 on the theory and applications of path integrals. It is the first book to explicitly solve path integrals of a wide variety of nontrivial quantum-mechanical systems, in particular the hydrogen atom. The solutions have become possible by two major advances. The first is a new euclidean path integral formula which increases the restricted range of applicability of Feynman's famous formula to include singular attractive 1/r and 1/r2 potentials. The second is a simple quantum equivalence principle governing the transformation of euclidean path integrals to spaces with curvature and torsion, which leads to time-sliced path integrals that are manifestly invariant under coordinate transformations. In addition to the time-sliced definition, the author gives a perturbative definition of path integrals which makes them invariant under coordinate transformations. A consistent implementation of this property leads to an extension of the theory of generalized functions by defining uniquely integrals over products of distributions. The powerful Feynman -- Kleinert variational approach is explained and developed systematically into a variational perturbation theory which, in contrast to ordinary perturbation theory, produces convergent expansions. The convergence is uniform from weak to strong couplings, opening a way to precise approximate evaluations of analytically unsolvable path integrals. Tunneling processes are treated in detail. The results are used to determine the lifetime of supercurrents, the stability of metastable thermodynamic phases, and the large-order behavior of perturbationexpansions. A new variational treatment extends the range of validity of previous tunneling theories from large to small barriers. A corresponding extension of large-order perturbation theory also applies now to small orders. Special attention is devoted to path integrals with topological restrictions. These are relevant to the understanding of the statistical properties of elementary particles and the entanglement phenomena in polymer physics and biophysics. The Chem-Simons theory of particles with fractional statistics (anyohs) is introduced and applied to explain the fractional quantum Hall effect. The relevance of path integrals to financial markets is discussed, and improvements of the famous Black -- Scholes formula for option prices are given which account for the fact that large market fluctuations occur much more frequently than in the commonly used Gaussian distributions.

Mathematical Theory of Feynman Path Integrals

Path Integrals on Group Manifolds

Gauge Fields